Example: A triangular port must be provided in the side of a form containing liquid concrete. Determine the resultant force and its point of application for the scenario below. The specific gravity of concrete is about 2.5. (376 N, 0.3 m)

\[
F_x = \frac{b a^3}{3a} \\
y_c = \frac{2}{3} a \\
A = \frac{1}{2} b a \\
y_{cp} = \frac{2}{3} a + \frac{F_x c}{y_c A}
\]

\[
y_{cp} = \frac{2}{3} a + \frac{b \alpha^3}{\frac{2}{3} a \left(\frac{1}{2} b a\right)} \frac{1}{3b}
\]

\[
y_{cp} = \frac{2}{3} a + \frac{1}{12} a = 0.3 m
\]
Example: The gate below is hinged at \( H \). The gate is 2 m wide normal to the page. Calculate the force required to keep the gate closed. (32.7 kN)

\[
I_{xc} = \frac{1}{12} (w)(2w)^3 \quad y_c = \frac{1.5}{2w} \quad A = (2w)(w)
\]

\[
y_{cp} = y_c + \frac{I_{xc}}{y_c A}
\]

\[
F(2w) = F_{woc}(y_c - y_{cp})
\]

\[
F_{res} = \frac{g g A \sin \theta}{2} = \frac{g A y_c 0.5}{2}
\]

\[
F = \frac{F_{res}}{2}
\]

\[
2M_0 = -F \cdot 2m + F_{res} y_{cp} = 0
\]

\[
F \cdot 2m = F_{res} y_{cp} = F_{res} \cdot 1
\]

\[
F = \frac{F_{res}}{2}
\]

\[
\theta = 150^\circ \text{ from free surface}
\]

\[
\tan \theta = \frac{a}{b}
\]

\[
a = \sin \theta (2m)
\]

\[
\frac{a}{2m} = \sin 50^\circ
\]
Example 2.6 – Consider the inclined gate below. What is the magnitude of the force exerted by the water on the gate and where does this force act?

\[ F_{res} = \rho g \sin \theta \int y \, dA \]

\[ = \rho g \sin \theta \int_0^2 2x' y \, dy' \]

\[ y = 6 - y' \]

\[ dy = -dy' \]

\[ x' = \left( \frac{y'}{4} \right)^{1/2} = \frac{\sqrt{y'}}{2} \]

\[ F_{res} = \frac{\rho g \sin 30^\circ \int_0^4 2y(6-y)^{1/2} \, dy}{2} \]

\[ = 599 \text{ lb} \]

\[ y_c = \frac{\int y^3 \, dA}{\int y \, dA} = \frac{\int y^2 (6-y)^{1/2} \, dy}{\int y (6-y)^{1/2} \, dy} = 3.9 \text{ ft} \]
A 4-m-long curved gate is located in the side of a reservoir containing water as shown in Fig. P2.70. Determine the magnitude of the horizontal and vertical components of the force of the water on the gate. Will this force pass through point A? Explain.

For equilibrium,
\[ \Sigma F_x = 0 \]
or
\[ F_H = F_2 = \gamma A_x = \gamma (6 \text{ m} + 1.5 \text{ m})(3 \text{ m} \times 4 \text{ m}) \]
so that
\[ F_H = (9.8 \text{ kN/m}^3)(25 \text{ m})(12 \text{ m}^2) = 882 \text{ kN} \]

Similarly,
\[ \Sigma F_y = 0 \]
\[ F_v = F_1 + q_w \text{ where } \]
\[ F_1 = \gamma A_w = (9.8 \text{ kN/m}^3)(6 \text{ m})(12 \text{ m}^2) \]
\[ q_w = \gamma \frac{A_w}{4} = (9.8 \text{ kN/m}^3)(9.4 \text{ m}^3) \]
Thus,
\[ F_v = (9.8 \text{ kN/m}^3) \left[ 72 \text{ m}^3 + 9 \pi \text{ m}^3 \right] = 983 \text{ kN} \]
(Note: Force of water on gate will be opposite in direction to that shown on figure.)

The direction of all differential forces acting on the curved surface is perpendicular to surface, and therefore, the resultant must pass through the intersection of all these forces which is at point A. Yes.
2.75 The concrete (specific weight = 150 lb/ft³) seawall of Fig. P.2.75 has a curved surface and restrains seawater at a depth of 24 ft. The trace of the surface is a parabola as illustrated. Determine the moment of the fluid force (per unit length) with respect to an axis through the toe (point A).

The components of the fluid force acting on the wall are \( F_1 \) and \( W \) as shown on the figure where:

\[
F_1 = \gamma h \rho A = (64,000 \text{ lb ft}^3)(24 \text{ ft})(24 \text{ ft} \times 1 \text{ ft}) = 18,400 \text{ lb} \quad \text{and} \quad y_1 = \frac{24 \text{ ft}}{3} = 8 \text{ ft}
\]

Also,

\[
W = \gamma V
\]

To determine \( V \), find area \( BCD \). Thus, (see figure to right)

\[
A = \int_0^{x_0} (24-y) \, dx = \int_0^{x_0} (24 - 0.2x^2) \, dx
\]

\[
= \left[ 24x - \frac{0.2x^3}{3} \right]_0^{x_0}
\]

and with \( x_0 = \sqrt{\frac{120}{a}} \), \( A = 175 \text{ ft}^2 \) so that

\[
V = A \times 1 \text{ ft} = 175 \text{ ft}^3
\]

Thus, \( W = (64,000 \text{ lb ft}^3)(175 \text{ ft}^3) = 11,200 \text{ lb} \)

To locate centroid of \( A \):

\[
x_c A = \int_0^{x_0} x \, dA = \int_0^{x_0} (24-y) \, dx = \int_0^{x_0} (24x - 0.2x^3) \, dx = 12x_0^2 - 0.2x_0^4
\]

and

\[
x_c = \frac{12 \left( \frac{120}{a} \right)^2 - 0.2 \left( \frac{120}{a} \right)^4}{175} = 4.11 \text{ ft}
\]

Thus,

\[
M_A = F_1 y_1 - W \left( 15 - x_c \right)
\]

\[
= (18,400 \text{ lb})(8 \text{ ft}) - (11,200 \text{ lb})(15 \text{ ft} - 4.11 \text{ ft}) = 25200 \text{ ft lb}
\]